

# Anti-Grand Unification and the Phase Transitions at the Planck Scale in Gauge Theories

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The way of the Grand unification of all interactions and the role of supersymmetry in GUTs are the problems of paramount importance in the contemporary elementary particle physics. However, at present time experiment doesn't indicate any manifestation of the Supersymmetry (see reviews [1-3]).

In this connection, the Anti-Grand Unification Theory (AGUT) was developed as a realistic alternative to SUSY GUTs by H.B.Nielsen (Niels Bohr Insitute, Denmark) and his collaborators: D.L.Bennett, C.D.Froggatt, L.V.Laperashvili, I.Picek and Y.Takanishi [4]-[17]. According to the AGUT, the supersymmetry doesn't come into existence up to the Planck energy scale:

$$\mu_{Pl} = 1.2 \cdot 10^{19} \text{ GeV}. \quad (1)$$

The SM is based on the group:

$$SMG = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y. \quad (2)$$

The AGUT suggests that at the scale  $\mu_G \lesssim \mu_{Pl}$  there exists the more fundamental group  $G$  containing  $N_{gen}$  copies of the Standard Model Group SMG:

$$G = SMG_1 \otimes SMG_2 \otimes \dots \otimes SMG_{N_{gen}} \equiv (SMG)^{N_{gen}} \quad (3)$$

where the integer  $N_{gen}$  designates the number of quark and lepton generations. If  $N_{gen} = 3$ , then the fundamental gauge group  $G$  is:

$$G = (SMG)^3 = SMG_1 \otimes SMG_2 \otimes SMG_3, \quad (4)$$

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or the generalized ones:

$$G_f = (SMG)^3 \otimes U(1)_f, \quad (5)$$

which is suggested by the fitting of fermion masses existing in the SM (see Refs. [9]-[12]) and

$$G_{ext} = (SMG \otimes U(1)_{B-L})^3, \quad (6)$$

which takes into account the see-saw mechanism with the right-handed neutrino [13],[14], also gives the reasonable fitting of the SM fermion masses and describes all neutrino experiments known today.

The group  $G_f$  contains the following gauge fields:  $3 \times 8 = 24$  gluons,  $3 \times 3 = 9$  W-bosons and  $3 \times 1 + 1 = 4$  abelian gauge bosons.

At first sight, this  $(SMG)^3 \times U(1)_f$  group with its 37 generators seems to be just one among many possible SM gauge group extensions. However, it is not such an arbitrary choice. There are at least reasonable requirements (postulates) on the gauge group  $G$  (or  $G_f$ , or  $G_{ext}$ ) which have uniquely to specify this group. It should obey the following postulates (the first two are also valid for SU(5) GUT):

1.  $G$  or  $G_f$  should only contain transformation transforming the known 45 Weyl fermions ( = 3 generations of 15 Weyl particles each) – counted as left handed, say – into each other unitarily, so that  $G$  (or  $G_f$ ) must be a subgroup of  $U(45)$ :  $G \subseteq U(45)$ .
2. No anomalies, neither gauge nor mixed. AGUT assumes that only straightforward anomaly cancellation takes place and forbids the Green-Schwarz type anomaly cancellation [18].
3. AGUT should NOT UNIFY the irreducible representations under the SM gauge group, called here SMG (see Eq.(2)).
4.  $G$  is the maximal group satisfying the above-mentioned postulates.

There are five Higgs fields named  $\phi_{WS}$ ,  $S$ ,  $W$ ,  $T$ ,  $\xi$  in the extended  $G_f$ -AGUT by Froggatt and Nielsen [11],[12]. These fields break the AGUT to the SM what means that their vacuum expectation values (VEV) are active. The field  $\phi_{WS}$  corresponds to the Weinberg–Salam theory,  $\langle S \rangle = 1$ , so that we have only three free parameters — three VEVs  $\langle W \rangle$ ,  $\langle T \rangle$  and  $\langle \xi \rangle$  to fit the experiment in the framework of this model. The authors of Refs.[11],[12] used them with aim to find the best fit to conventional experimental data for all fermion masses and mixing angles in the SM (see Table I). The result presented by Table I is encouraging. The fit is given by the  $\chi^2$  function (called here  $\tilde{\chi}^2$ ). The lowest value of  $\tilde{\chi}^2 (\approx 1.87)$  gives the following VEVs:

$$\langle S \rangle = 1; \quad \langle W \rangle = 0.179; \quad \langle T \rangle = 0.071; \quad \langle \xi \rangle = 0.099. \quad (7)$$

The extended Anti-GUT theory by Nielsen and Takanishi [13],[14], which is described by the group of symmetry  $G_{ext}$  (see Eq.(6)), was suggested with aim to explain the neutrino oscillations. Introducing the right-handed neutrino in the model, the authors replaced the assumption 1 and considered  $U(48)$  group instead of  $U(45)$ , so that  $G_{ext}$  is a subgroup of  $U(48)$ :  $G_{ext} \subseteq U(48)$ .

This group ends up having 7 Higgs fields falling into 4 classes according to the order of magnitude of the expectation values:

1) The smallest VEV Higgs field plays role of the SM Weinberg–Salam Higgs field  $\phi_{WS}$  having the weak scale value  $\langle \phi_{WS} \rangle = 246 \text{ GeV}/\sqrt{2}$ .

2) The next smallest VEV Higgs field breaks all families  $U(1)_{(B-L)}$  group, which is broken at the see–saw scale. This VEV is  $\langle \phi_{(B-L)} \rangle \sim 10^{12} \text{ GeV}$ . Such a field is absent in the "old" extended AGUT.

3) The next 4 Higgs fields are  $W$ ,  $T$ ,  $\xi$  and  $\chi$ , which have VEVs of the order of a factor 10 to 50 under the Planck unit. That means that if intermediate propagators have scales given by the Planck scale, as it is assumed in the AGUT in general, then they will give rise to suppression factors of the order 1/10 each time they are needed to cause a transition. The field  $\chi$  is absent in the "old"  $G_f$ –AGUT. It was introduced in Refs.[13],[14] for the purpose of the study neutrinos.

4) The last one, with VEV of the same order as the Planck scale, is the Higgs field  $S$ . It had VEV  $\langle S \rangle = 1$  in the "old" extended AGUT by Froggatt and Nielsen (with  $G_f$  group of symmetry), but this VEV is not equal to unity in the "new" extended AGUT. Therefore there is a possibility to observe phenomenological consequences of the field  $S$  in the Nielsen–Takanishi model.

Typical fit to the masses and mixing angles for the SM leptons and quarks in the framework of the  $G_{ext}$ –AGUT is given by Table II. The lowest value of  $\tilde{\chi}^2$  is  $\approx 1.46$ . In contrast to the "old" extended AGUT, the new results are more encouraging.

The AGUT approach is used in conjunction with the Multiple Point Principle (MPP) proposed by D.L.Bennett and H.B.Nielsen [6]. According to this principle Nature seeks a special point — the Multiple Critical Point (MCP) — which is a point on the phase diagram of the fundamental regularized gauge theory  $G$  (or  $G_f$ , or  $G_{ext}$ ), where the vacua of all fields existing in Nature are degenerate having the same vacuum energy density. This is the Multiple Point Principle. Such a phase diagram has axes given by all coupling constants considered in theory. Then all (or just many) numbers of phases meet at the MCP.

In the AGUT at some point  $\mu_G$  the group  $G$  (or  $G_f$ , or  $G_{ext}$ ) undergoes spontaneous breakdown to the diagonal subgroup:

$$G \longrightarrow G_{diag.subgr.} = \{g, g, g | g \in SMG\}, \quad (8)$$

which is identified with the usual (lowenergy) group SMG.

Multiple Point Model assumes the existence of MCP at the Planck scale, insofar as gravity may be "critical" at the Planck scale.

The idea of MPP has its origin from the lattice investigations of gauge theories. In particular, Monte Carlo simulations on lattice of  $U(1)$ -,  $SU(2)$ - and  $SU(3)$ - gauge theories indicate the existence of a triple (critical) point, which is a boundary point of three first order phase transitions. Using the Monte Carlo results on lattice, it is possible to make theoretical calculations of the critical coupling constants and obtain slightly more accurate predictions of the AGUT for the SM fine structure constants.

In the SM the usual definition of coupling constants is used:

$$\alpha_1 = \frac{5}{3} \frac{\alpha}{\cos^2 \Theta_{\overline{MS}}}, \quad \alpha_2 = \frac{\alpha}{\sin^2 \Theta_{\overline{MS}}}, \quad \alpha_3 \equiv \alpha_s = \frac{g_s^2}{4\pi}, \quad (9)$$

where  $\alpha$  and  $\alpha_s$  are the electromagnetic and strong fine structure constants, respectively. All values are defined in the Modified minimal subtraction scheme ( $\overline{MS}$ ). Using experimentally given parameters and the renormalization group equations (RGE), it is possible to extrapolate the experimental values of three inverse running constants  $\alpha_i^{-1}$  (here  $i = 1, 2, 3$  corresponds to U(1), SU(2) and SU(3) groups) from Electroweak scale to the Planck scale. The precision of the LEP data allows to make this extrapolation with small errors. Assuming that the RGEs are contingent not encountering new particles up to  $\mu \lesssim \mu_{Pl}$  and doing the extrapolation with one Higgs doublet under the assumption of a "desert", the following result for the inverses  $\alpha_{Y,2,3}^{-1}$  ( $\alpha_Y \equiv \frac{3}{5}\alpha_1$ ) was obtained in Ref.[6]:

$$\alpha_Y^{-1}(\mu_{Pl}) \approx 55.5; \quad \alpha_2^{-1}(\mu_{Pl}) \approx 49.5; \quad \alpha_3^{-1}(\mu_{Pl}) \approx 54. \quad (10)$$

The extrapolation of  $\alpha_i^{-1}(\mu)$  up to the point  $\mu = \mu_G$  is shown in Fig.1. The AGUT predicts their values at the scale  $\mu_G \sim 10^{18}$  GeV (which is very close to  $\mu_{MCP} = \mu_{Pl}$ ) in terms of the critical couplings  $\alpha_{i,crit}$  taken from the lattice gauge theory [19]-[23]:

$$\alpha_i(\mu_{Pl}) = \frac{\alpha_{i,crit}}{N_{gen}} = \frac{\alpha_{i,crit}}{3} \quad (11)$$

for  $i=2,3$  and

$$\alpha_1(\mu_{Pl}) = \frac{\alpha_{1,crit}}{\frac{1}{2}N_{gen}(N_{gen} + 1)} = \frac{\alpha_{1,crit}}{6} \quad (12)$$

for U(1).

According to the AGUT, at the Planck scale the running constants  $\alpha_1$  (or  $\alpha_Y \equiv \frac{3}{5}\alpha_1$ ),  $\alpha_2$  and  $\alpha_3$ , as chosen by Nature, are just the ones corresponding to the MCP.

There exists a simple explanation of the relations (11) and (12). As it was mentioned above, the group G breaks down at  $\mu = \mu_G$ . It should be said that at the very high energies  $\mu \geq \mu_G \lesssim \mu_{Pl}$  (see Fig.1) each generation has its own gluons, own W's etc. The breaking makes only linear combination of a certain color combination of gluons which exists below  $\mu = \mu_G$  and down to the low energies. We can say that the phenomenological gluon is a linear combination (with amplitude  $1/\sqrt{3}$  for  $N_{gen} = 3$ ) for each of the AGUT-gluons of the same color combination. This means that coupling constant for the phenomenological gluon has a strength that is  $\sqrt{3}$  times smaller, if as we effectively assume that three AGUT SU(3) couplings are equal to each other. Then we have the following formula connecting the fine structure constants of G-theory (e.g. AGUT) and low energy surviving diagonal subgroup  $G_{diag.subg.} \subseteq (SMG)^3$  given by Eq.(8):

$$\alpha_{diag,i}^{-1} = \alpha_{1st\ gen.,i}^{-1} + \alpha_{2nd\ gen.,i}^{-1} + \alpha_{3rd\ gen.,i}^{-1}. \quad (13)$$

Here  $i = U(1), SU(2), SU(3)$ , and  $i=3$  means that we talk about the gluon couplings. For non-Abelian theories we immediately obtain Eq.(11) from Eq.(13) at the critical point (MCP).

In contrast to non-Abelian theories, in which the gauge invariance forbids the mixed (in generations) terms in the Lagrangian of G-theory, the  $U(1)$ -sector of the AGUT contains such mixed terms what explains the difference between the expressions (11) and (12).

Using Monte Carlo results on lattice the AGUT predicts [6] :

$$\alpha_Y^{-1}(\mu_{MCP}) = 55 \pm 6, \quad \alpha_2^{-1}(\mu_{MCP}) = 49.5 \pm 3, \quad \alpha_3^{-1}(\mu_{MCP}) = 57 \pm 3, \quad (14)$$

in correspondence with the result (10).

According to Eq.(12), the first values of Eqs.(10) and (13) gives the following estimation for the  $U(1)$  fine structure constant at  $\mu = \mu_{MCP}$ :

$$\alpha_{crit}^{-1} \sim 9. \quad (15)$$

The Monte Carlo simulations of the lattice  $U(1)$  gauge theory gives [20]-[22] :

$$\alpha_{crit} \approx 0.20 \pm 0.015, \quad \text{or} \quad \alpha_{crit}^{-1} \approx 5. \quad (16)$$

However, it is possible to show (see [6]) that quantum fluctuations encrease the value of  $\alpha_{crit}^{-1}$  giving the value (15).

The following hypothesis was stated in Refs.[16],[17]: it is possible that the existence of monopoles at superhigh energies — at a short distance from the Planck energy scale (1) — plays an essential role in the phase transitions at the Planck scale when all fields existing in the SM turn into the new phase, say, (super)string phase. In the previous works [6],[7],[15] the investigation of the phase transition phenomena and, in particular, the calculation of the  $U(1)$  critical coupling constant were connected with the existence of artifact monopoles in the lattice gauge theory and also in the Wilson loop action model, which we proposed in Ref.[15]. Now, instead of using the lattice or Wilson loop cut-off, we are going to introduce physically existing monopoles into the theory as fundamental fields.

Developing a version of the local field theory of the Higgs scalar monopoles and electrically charged particles, we consider an Abelian gauge theory in the Zwanziger formalism [24]-[28] and look for a/or rather several phase transitions connected with the monopoles forming a condensate in the vacuum.

The Zwanziger formalism [24],[25] (see also [26],[27] and review [28]) considers two potentials  $A_\mu(x)$  and  $B_\mu(x)$  describing one physical photon with two physical degrees of freedom. Now and below we call this theory QEMD ("Quantum ElectroMagnetodynamics").

In QEMD the total field system of the gauge, electrically ( $\Psi$ ) and magnetically ( $\Phi$ ) charged fields (with charges  $e$  and  $g$ , respectively) is described by the partition function which has the following form in the Euclidean space:

$$Z = \int [DA][DB][D\Phi][D\Phi^+][D\Psi][D\Psi^+] e^{-S}, \quad (17)$$

where

$$S = \int d^4x L(x) = S_{Zw}(A, B) + S_{gf} + S_{(matter)}. \quad (18)$$

The Zwanziger action  $S_{Zw}(A, B)$  is given by:

$$S_{Zw}(A, B) = \int d^4x \left[ \frac{1}{2} (n \cdot [\partial \wedge A])^2 + \frac{1}{2} (n \cdot [\partial \wedge B])^2 + \right. \\ \left. + \frac{i}{2} (n \cdot [\partial \wedge A]) (n \cdot [\partial \wedge B]^*) - \frac{i}{2} (n \cdot [\partial \wedge B]) (n \cdot [\partial \wedge A]^*) \right], \quad (19)$$

where we have used the following designations:

$$[A \wedge B]_{\mu\nu} = A_\mu B_\nu - A_\nu B_\mu, \quad (n \cdot [A \wedge B])_\mu = n_\nu (A \wedge B)_{\nu\mu}, \\ G_{\mu\nu}^* = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} G_{\lambda\rho}. \quad (20)$$

In Eqs.(19) and(20) the unit vector  $n_\mu$  represents the fixed direction of the Dirac string in the 4-space.

The action  $S_{(matter)} = \int d^4x L_{(matter)}(x)$  describes the electrically and magnetically charged matter fields.  $S_{gf}$  is the gauge-fixing action (see [26]).

Let us consider now the Lagrangian  $L_{(matter)}$  describing the Higgs scalar fields  $\Psi(x)$  and  $\Phi(x)$  interacting with gauge fields  $A_\mu(x)$  and  $B_\mu(x)$ , respectively:

$$L_{(matter)}(x) = \frac{1}{2} |D_\mu \Psi|^2 + \frac{1}{2} |\tilde{D}_\mu \Phi|^2 - U(\Psi, \Phi), \quad (21)$$

where

$$D_\mu = \partial_\mu - ieA_\mu, \quad \text{and} \quad \tilde{D}_\mu = \partial_\mu - igB_\mu \quad (22)$$

are covariant derivatives;

$$U(\Psi, \Phi) = \frac{1}{2} \mu_e^2 |\Psi|^2 + \frac{\lambda_e}{4} |\Psi|^4 + \frac{1}{2} \mu_m^2 |\Phi|^2 + \frac{\lambda_m}{4} |\Phi|^4 + \lambda_1 |\Psi|^2 |\Phi|^2 \quad (23)$$

is the Higgs potential for the electrically and magnetically charged fields  $\Psi$  and  $\Phi$ . The complex scalar fields:

$$\Psi = \psi + i\zeta \quad \text{and} \quad \Phi = \phi + i\chi \quad (24)$$

contain the Higgs  $(\psi, \phi)$  and Goldstone  $(\zeta, \chi)$  boson fields.

The Lorentz invariance is lost in the Zwanziger Lagrangian (19) because of depending on a fixed vector  $n_\mu$ , but this invariance regained for the quantized values of coupling constants  $e$  and  $g$  obeying the Dirac relation:

$$e_i g_j = 2\pi n_{ij}, \quad n_{ij} \in \mathbb{Z}. \quad (25)$$

Considering the electric and magnetic fine structure constants:  $\alpha = \frac{e^2}{4\pi}$  and  $\tilde{\alpha} = \frac{g^2}{4\pi}$  we have the invariance of the QEMD under the interchange  $\alpha \leftrightarrow \tilde{\alpha}$ .

For  $n_{ij} = 1$  from the Dirac relation (25) we have:

$$\alpha \tilde{\alpha} = \frac{1}{4}. \quad (26)$$

The effective potential in the Higgs model of electrodynamics for a charged scalar field was calculated in the one-loop approximation for the first time by the authors of Ref.[29] (see also review [30]). Using this method we can construct the effective potential (also in the one-loop approximation) for the theory described by the partition function (17) with the action  $S$ .

Let us consider now the shifts:  $\Psi = \Psi_B + \hat{\Psi}(x)$ ,  $\Phi(x) = \Phi_B + \hat{\Phi}(x)$  with  $\Psi_B$  and  $\Phi_B$  as background fields and calculate the following expression for the partition function in the one-loop approximation:

$$\begin{aligned} Z &= \int [DA][DB][D\hat{\Phi}][D\hat{\Phi}^+][D\hat{\Psi}][D\hat{\Psi}^+] \times \\ &\exp\{-S(A, B, \Phi_B, \Psi_B) - \int d^4x [\frac{\delta S(\Phi)}{\delta \Phi(x)}|_{\Phi=\Phi_B} \hat{\Phi}(x) + \frac{\delta S(\Psi)}{\delta \Psi(x)}|_{\Psi=\Psi_B} \hat{\Psi}(x) + h.c.]\} \\ &= \exp\{-F(\Psi_B, \Phi_B, e^2, g^2, \mu_e^2, \mu_m^2, \lambda_e, \lambda_m)\}. \end{aligned} \quad (27)$$

Using the representations (24), we obtain the effective potential:

$$V_{eff} = F(\psi_B, \phi_B, e^2, g^2, \mu_e^2, \mu_m^2, \lambda_e, \lambda_m) \quad (28)$$

given by the function  $F$  of Eq.(27) for the constant background fields:  $\Psi_B = \psi_B = \text{const}$ ,  $\Phi_B = \phi_B = \text{const}$ .

The effective potential (28) has several minima. Their position depends on  $e^2, g^2, \mu_{e,m}^2$  and  $\lambda_{e,m}$ . If the first local minimum occurs at  $\psi_B = 0$  and  $\phi_B = 0$ , it corresponds to the the Coulomb-like phase in our description.

We are interested in the phase transition from the Coulomb-like phase " $\psi_B = \phi_B = 0$ " to the confinement phase " $\psi_B = 0, \phi_B = \phi_0 \neq 0$ ". In this case the one-loop effective potential for monopoles coincides with the expression of the effective potential calculated by authors of Ref.[29] for scalar electrodynamics and extended to the massive theory in Ref.[31].

Assuming the existence of the first vacuum at  $\phi_B = 0$  and using from now the designations:  $\mu = \mu_m, \lambda = \lambda_m$ , we have the effective potential in the Higgs monopole model described by the following expression equivalent to that considered in Ref.[29]:

$$V_{eff}(\phi^2) = \frac{\mu_{run}^2}{2} \phi_B^2 + \frac{\lambda_{run}}{4} \phi_B^4 + \frac{\mu^4}{64\pi^2} \log \frac{(\mu^2 + 3\lambda\phi_B^2)(\mu^2 + \lambda\phi_B^2)}{\mu^4}, \quad (29)$$

where

$$\lambda_{run}(\phi_B^2) = \lambda + \frac{1}{16\pi^2} [3g^4 \log \frac{\phi_B^2}{M^2} + 9\lambda^2 \log \frac{\mu^2 + 3\lambda\phi_B^2}{M^2} + \lambda^2 \log \frac{\mu^2 + \lambda\phi_B^2}{M^2}], \quad (30)$$

$$\mu_{run}^2(\phi_B^2) = \mu^2 + \frac{\lambda\mu^2}{16\pi^2} [3 \log \frac{\mu^2 + 3\lambda\phi_B^2}{M^2} + \log \frac{\mu^2 + \lambda\phi_B^2}{M^2}]. \quad (31)$$

Here  $M$  is the cut-off scale.

As it was shown in Ref.[29], the one-loop effective potential (29) can be improved by the consideration of the renormalization group equation (RGE).

According to Refs.[29]-[31], RGE for the improved one-loop effective potential is given by the following expression:

$$(M^2 \frac{\partial}{\partial M^2} + \beta_\lambda \frac{\partial}{\partial \lambda} + \beta_g \frac{\partial}{\partial g} + \beta_{(\mu^2)} \mu^2 \frac{\partial}{\partial \mu^2} - \gamma \phi^2 \frac{\partial}{\partial \phi^2}) V_{eff}(\phi^2) = 0, \quad (32)$$

where the function  $\gamma$  is the anomalous dimension:  $\gamma(\frac{\phi}{M}) = -\frac{\partial \phi}{\partial M}$ . The  $\gamma$ -expression for monopoles is given by Ref.[29] with replacement  $e \rightarrow g$ :

$$\gamma = -\frac{3g_{run}^2}{16\pi^2}. \quad (33)$$

RGE (32) leads to a new improved effective potential:

$$V_{eff}(\phi^2) = \frac{1}{2} \mu_{run}^2(t) G^2(t) \phi^2 + \frac{1}{4} \lambda_{run}(t) G^4(t) \phi^4, \quad (34)$$

where

$$G(t) \equiv \exp[-\frac{1}{2} \int_0^t dt' \gamma(g_{run}(t'), \lambda_{run}(t'))] \quad \text{with} \quad t = \log(\phi^2/M^2). \quad (35)$$

Let us write now the one-loop potential (29) as

$$V_{eff} = V_0 + V_1, \quad \text{where} \quad V_0 = \frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4, \quad (36)$$

$$V_1 = \frac{1}{64\pi^2} [3g^4 \phi^4 \log \frac{\phi^2}{M^2} + (\mu^2 + 3\lambda \phi^2)^2 \log \frac{\mu^2 + 3\lambda \phi^2}{M^2} + (\mu^2 + \lambda \phi^2)^2 \log \frac{\mu^2 + \lambda \phi^2}{M^2} - 2\mu^4 \log \frac{\mu^2}{M^2}]. \quad (37)$$

We can plug this  $V_{eff}$  into RGE (32) and obtain the following RG-equations (see [30]):

$$\frac{d\lambda_{run}}{dt} = \frac{1}{16\pi^2} (3g_{run}^4 + 10\lambda_{run}^2 - 6\lambda_{run}g_{run}^2), \quad (38)$$

$$\frac{d\mu_{run}^2}{dt} = \frac{\mu_{run}^2}{16\pi^2} (4\lambda_{run} - 3g_{run}^2). \quad (39)$$

The Dirac relation and the RGE for the electric and magnetic fine structure constants  $\alpha$  and  $\tilde{\alpha}$  were investigated in detail in the recent paper [27]. The following result was obtained.

If we have the electrically and magnetically charged particles existing simultaneously for  $\tilde{\mu} > \tilde{\mu}_{(threshold)}$  and if in some region of  $\tilde{\mu}$  their  $\beta$ -functions are computable perturbatively as a power series in  $e^2$  and  $g^2$ , then the Dirac relation is valid not only for the "bare" elementary charges  $e_0$  and  $g_0$ , but also for the renormalized effective charges  $e$  and  $g$  (see [32] and review [28]), and the following RGEs (obtained in Ref.[27]) take place:

$$\frac{d \log \alpha(p)}{dt} = -\frac{d \log \tilde{\alpha}(p)}{dt} = \beta^{(e)}(\alpha) - \beta^{(m)}(\tilde{\alpha}). \quad (40)$$



These RGEs are in accordance with the Dirac relation (26) and the dual symmetry considered above. By restricting ourselves to the two-loop approximation for  $\beta$ -functions, we have the following equations (40) for scalar particles:

$$\frac{d \log \alpha(p)}{dt} = -\frac{d \log \tilde{\alpha}(p)}{dt} = \frac{\alpha - \tilde{\alpha}}{12\pi} \left(1 + 3\frac{\alpha + \tilde{\alpha}}{4\pi} + \dots\right). \quad (41)$$

According to Eq.(41), the two-loop contribution is not more than 30% if both  $\alpha$  and  $\tilde{\alpha}$  obey the following requirement:

$$0.25 \lesssim \alpha, \tilde{\alpha} \lesssim 1. \quad (42)$$

The lattice simulations of compact QED give the behavior of the effective fine structure constant  $\alpha(\beta)$  ( $\beta = 1/e_0^2$ , and  $e_0$  is the bare electric charge) in the vicinity of the phase transition point (see Refs.[20],[22]). The following critical values of the fine structure constant  $\alpha$  and  $\tilde{\alpha}$  was obtained in Ref.[22]:

$$\alpha_{crit}^{lat} \approx 0.20 \pm 0.015 \quad \tilde{\alpha}_{crit}^{lat} \approx 1.25 \pm 0.10 \quad \text{at} \quad \beta_{crit} \approx 1.011. \quad (43)$$

These values almost coincide with the borders of the requirement (42) given by the perturbation theory for  $\beta$ -functions.

In the one-loop approximation, we have:

$$\frac{dg_{run}^2}{dt} = \frac{g_{run}^4}{48\pi^2} - \frac{1}{12}. \quad (44)$$

Here the second term describes the influence of the electrically charged fields on the behavior of the monopole charge.

Investigating the phase transition from the Coulomb-like phase " $\psi_B = \phi_B = 0$ " to the phase with " $\psi_B = 0, \phi_B = \phi_0 \neq 0$ ", we see that the effective potential (34) has the first and the second minima appearing at  $\phi = 0$  and  $\phi = \phi_0$ , respectively. They are shown in Fig.2 by the curve "1". These minima of  $V_{eff}(\phi^2)$  correspond to the different vacua arising in the model.

The conditions for the existence of degenerate vacua are given by the following requirements:

$$V_{eff}(0) = V_{eff}(\phi_0^2) = 0, \quad (45)$$

$$V'_{eff}(\phi_0^2) \equiv \frac{\partial V_{eff}}{\partial \phi^2} \Big|_{\phi=\phi_0} = 0, \quad (46)$$

$$V''_{eff}(\phi_0^2) \equiv \frac{\partial^2 V_{eff}}{\partial (\phi^2)^2} \Big|_{\phi=\phi_0} > 0. \quad (47)$$

From the first equation (45) applied to Eq.(34) we have:

$$\mu_{run}^2 = -\frac{1}{2}\lambda_{run}(t_0)\phi_0^2 G^2(t_0), \quad \text{where} \quad t_0 = \log(\phi_0^2/M^2). \quad (48)$$

The joint solution of equations  $V_{eff}(\phi_0^2) = V'_{eff}(\phi_0^2) = 0$  gives:

$$g_{crit}^4 = -2\lambda_{run}\left(\frac{8\pi^2}{3} + \lambda_{run}\right). \quad (49)$$

The curve (49) is represented on the phase diagram  $(\lambda_{run}; g_{run}^4)$  of Fig.3 by the curve "1" which describes a border between the "Coulomb-like" phase with  $V_{eff} \geq 0$  and the confinement ones having  $V_{eff}^{min} < 0$ .

The next step is the calculation of the second derivative of the effective potential. Let us consider now the case when this second derivative changes its sign giving a maximum of  $V_{eff}$  instead of the minimum at  $\phi^2 = \phi_0^2$ . Such a possibility is shown in Fig.2 by the dashed curve "2". Now two additional minima at  $\phi^2 = \phi_1^2$  and  $\phi^2 = \phi_2^2$  appear in our theory. They correspond to two different confinement phases related with the confinement of the electrically charged particles. If these two minima are degenerate, then we have the following requirements:

$$V_{eff}(\phi_1^2) = V_{eff}(\phi_2^2) < 0, \quad V'_{eff}(\phi_1^2) = V'_{eff}(\phi_2^2) = 0, \quad (50)$$

which describe the border between the confinement phases "conf.1" and "conf.2" presented in Fig.3 by curve "3". This curve "3" meets the curve "1" at the triple point A. According to the illustration shown in Fig.3, it is obvious that this triple point A is given by the following requirements:

$$V_{eff}(\phi_0^2) = V'_{eff}(\phi_0^2) = V''_{eff}(\phi_0^2) = 0. \quad (51)$$

In contrast to the requirements:

$$V_{eff}(\phi_0^2) = V'_{eff}(\phi_0^2) = 0, \quad (52)$$

producing the curve "1", let us consider now the joint solution of the following equations:

$$V_{eff}(\phi_0^2) = V''_{eff}(\phi_0^2) = 0. \quad (53)$$

The dashed curve "2" of Fig.3 represents the solution of Eq.(53). This curve is going very close to the maximum of the curve "1". It is natural to assume that the position of the triple point A coincides with this maximum and the corresponding deviation can be explained by our approximate calculations. Taking into account such an assumption, let us consider the border between the phase "conf.1" having the first minimum at nonzero  $\phi_0$  with  $V_{eff}^{min} = c_1 < 0$  and the phase "conf.2" which reveals two minima with the second minimum being the deeper one and having  $V_{eff}^{min} = c_2 < 0$ . This border (described by the curve "3" of Fig.3) was calculated in the vicinity of the triple point A by means of Eqs.(50) with  $\phi_1$  and  $\phi_2$  represented as  $\phi_{1,2} = \phi_0 \pm \epsilon$  with  $\epsilon \ll \phi_0$ . The result of such calculations gives the following expression for the curve "3":

$$g_{run}^4 = \frac{5}{2}(5\lambda_{run} + 8\pi^2)\lambda_{run} + 8\pi^4. \quad (54)$$

The piece of the curve "1" to the left of the point A describes the border between the "Coulomb-like" phase and the phase "conf.1". The right piece of the curve "1" along to the right of the point B separates the "Coulomb" phase and the phase "conf.2". But between the points A and B the phase transition border is going slightly upper the curve "1". This deviation is very small and can't be distinguished on Fig.3.

The numerical solution demonstrates that the triple point A exists in the very neighborhood of the maximum of the curve (49) and its position is approximately given by the following values:

$$\lambda_{(A)} \approx -\frac{4\pi^2}{3} \approx -13.4, \quad (55)$$

$$g_{(A)}^2 = g_{crit}^2|_{\text{for } \lambda_{run}=\lambda_{(A)}} \approx \frac{4\sqrt{2}}{3}\pi^2 \approx 18.6. \quad (56)$$

The triple point value of the electric and magnetic fine structure constant follow from Eqs.(26) and (56):

$$\alpha_{(A)} = \frac{\pi}{g_{(A)}^2} \approx 0.17, \quad \tilde{\alpha}_{(A)} = \frac{g_{(A)}^2}{4\pi} \approx 1.48. \quad (57)$$

The obtained result is very close to the Monte Carlo lattice result (43).

The phase diagram drawn in Fig.3 corresponds to the validity of one-loop approximation (with accuracy of deviations not more than 30%, see Ref.[17]) in the region of parameters:

$$0.17 \lesssim \alpha, \tilde{\alpha} \lesssim 1.5, \quad (58)$$

It is necessary to note that the RGE for  $\lambda_{run}$  indicates a slow convergence of the series over  $\lambda$  (see Ref.[33]) and the one-loop approximation is valid for  $\lambda_{run}$  up to  $|\lambda| \lesssim 30$  with accuracy of deviations  $< 10\%$ .

It is obvious that in our case both phases, "conf.1" and "conf.2", have nonzero monopole condensate in the minima of the effective potential, when  $V_{eff}^{min}(\phi_{1,2} \neq 0) < 0$ . By this reason, the Abrikosov–Nielsen–Olesen (ANO) electric vortices [34],[35] may exist in these both phases, which are equivalent in the sense of the "string" formation. If electric charges are present in a model, they are placed at the ends of the vortices—"strings" and therefore are confined. The phase diagram of Fig.3 demonstrates the existence of the confinement phase for  $\alpha \geq \alpha_{(A)} \approx 0.17$ .

The lattice investigations [20],[22] show that in the confinement phase  $\alpha(\beta)$  increases when  $\beta = 1/e_0^2 \rightarrow 0$  (here  $e_0$  is the bare electric charge) and very slowly approaches to its maximal value:  $\alpha_{max} = \frac{\pi}{12} \approx 0.26$  predicted in Ref.[36] (see also Ref.[15]).

It is worthwhile mentioning that the confinement of monopoles can be described by using duality. The Higgs field  $\Psi$ , having the electric charge, is responsible for this confinement. The corresponding confinement phases for monopoles are absent on the phase diagram of Fig.3. They can be described by the phase diagram  $(\lambda_{run}^e; e_{run}^2)$ . The overall phase diagram is three-dimensional and is given by  $(\lambda_{run}^m; \lambda_{run}^e; g_{run}^2)$  ( $e_{run}^2$  and  $g_{run}^2$  are related by the Dirac relation).

The result (64) obtained in the framework of the Higgs scalar monopole model gives the following prediction:

$$\alpha_{crit}^{-1} = \alpha_{(A)}^{-1} \approx 6, \quad (59)$$

which is comparable with the MPM result (6).

Although the one-loop approximation for the (improved) effective potential does not give an exact coincidence with the MPM prediction of the critical  $\alpha$ , we see that, in general, the Higgs monopole model is very encouraging for the AGUT-MPM. We have a hope that the two-loop approximation corrections to the Coleman-Weinberg effective potential will lead to the better accuracy in calculation of the phase transition couplings.

The review of all existing results gives:

1)

$$\alpha_{crit}^{lat} \approx 0.20 \pm 0.015, \quad \tilde{\alpha}_{crit}^{lat} \approx 1.25 \pm 0.10 \quad (60)$$

– in the Compact QED with the Wilson lattice action [20];

2)

$$\alpha_{crit}^{lat} \approx 0.204 \quad \tilde{\alpha}_{crit}^{lat} \approx 1.25 \quad (61)$$

– in the model with the Wilson loop action [15];

3)

$$\alpha_{crit} \approx 0.1836, \quad \tilde{\alpha}_{crit} \approx 1.36 \quad (62)$$

– in the Compact QED with the Villain lattice action [22];

4)

$$\alpha_{crit} = \alpha_{(A)} \approx 0.17, \quad \tilde{\alpha}_{crit} = \tilde{\alpha}_{(A)} \approx 1.48 \quad (63)$$

– in the Higgs scalar monopole model (the present paper).

Hereby we see an additional arguments for our previously hoped (see [15] and [16]) "approximate universality" of the first order phase transition couplings: the fine structure constant (in the continuum) is at the/a multiple point approximately the same one independent of various parameters of the different (lattice, etc.) regularization.

Recent investigations (L.V.Laperashvili, H.B.Nielsen, Bled Workshop, Slovenia, 2000) show that monopoles are confined in the SM up to the Planck scale. But they can exist in the AGUT. Let us assume now the existence of monopoles at superhigh energies  $\mu \geq \mu_{mon} \lesssim \mu_{Pl}$ . Then RGEs (40) describing also monopoles can lead to the Unification of all interactions at the Planck scale giving the coincidence of all  $\alpha_{i,crit}$  at the point  $\mu \sim \mu_{Pl}$ . Such a situation is shown in Fig.1 by dashed curves. These investigations are in progress.

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**Table I:** Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass  $M_t$  which is the pole mass.

	Fitted	Experimental
$m_u$	3.6 MeV	4 MeV
$m_d$	7.0 MeV	9 MeV
$m_e$	0.87 MeV	0.5 MeV
$m_c$	1.02 GeV	1.4 GeV
$m_s$	400 MeV	200 MeV
$m_\mu$	88 MeV	105 MeV
$M_t$	192 GeV	180 GeV
$m_b$	8.3 GeV	6.3 GeV
$m_\tau$	1.27 GeV	1.78 GeV
$V_{us}$	0.18	0.22
$V_{cb}$	0.018	0.041
$V_{ub}$	0.0039	0.0035

**Table II:** Best fit to conventional experimental data in the "new" extended AGUT. All masses are running masses at 1 GeV except the top quark mass  $M_t$  which is the pole mass.

	Fitted	Experimental
$m_u$	3.1 MeV	4 MeV
$m_d$	6.6 MeV	9 MeV
$m_e$	0.76 MeV	0.5 MeV
$m_c$	1.29 GeV	1.4 GeV
$m_s$	390 MeV	200 MeV
$m_\mu$	85 MeV	105 MeV
$M_t$	179 GeV	180 GeV
$m_b$	7.8 GeV	6.3 GeV
$m_\tau$	1.29 GeV	1.78 GeV
$V_{us}$	0.21	0.22
$V_{cb}$	0.023	0.041
$V_{ub}$	0.0050	0.0035

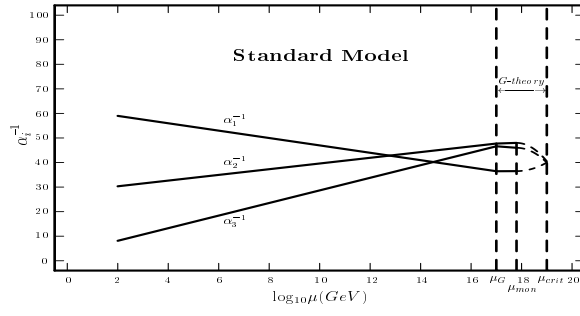


Figure 1.

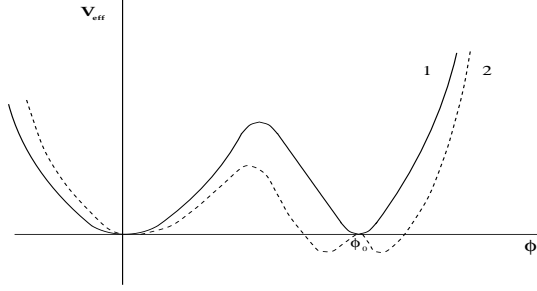


Fig. 2  
The effective potential  $V_{eff}$ : the curve 1 corresponds to the "Coulomb"-  
"confinement" phase transition; curve 2 describes the existence of two  
minima corresponding to the confinement phases.

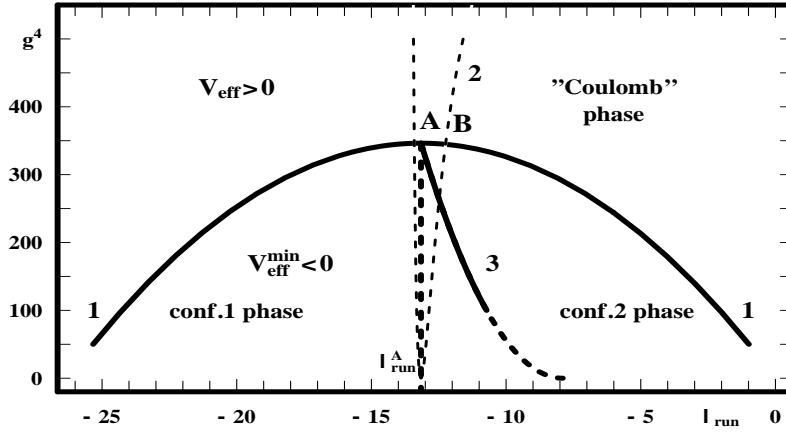


Figure 3: The phase diagram  $(\lambda_{run}; g^4 \equiv g_{run}^4)$  corresponding to the Higgs  
monopole model shows the existence of a triple point A  $(\lambda_{(A)} \approx -13.4; g_{(A)}^2 \approx$   
18.6).